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$B \to K^* \gamma$ FROM HYBRID SUM RULE

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Abstract

Using the *hybrid* moments-Laplace sum rule (HSR), which is well-defined for $M_b \to \infty$, in contrast with the *popular* double Borel (Laplace) sum rule (DLSR), which blows up in this limit when applied to the heavy-to-light processes, we show that the form factor of the $B \to K^* \gamma$ radiative transition is dominated by the light-quark condensate for $M_b \to \infty$ and behaves like $\sqrt{M_b}$. The form factor is found to be $F_1^{B\to K^*}(0) \simeq (30.8 \pm 1.3 \pm 3.6 \pm 0.6) \times 10^{-2}$, where the errors come respectively from the procedure in the sum rule analysis, the errors in the input and in the $SU(3)_f$-breaking parameters. This result leads to $Br(B \to K^* \gamma) \simeq (4.45 \pm 1.12) \times 10^{-5}$ in agreement with the recent CLEO data. Parametrization of the M_b-dependence of the form factor including the $SU(3)_f$-breaking effects is given in (26), which leads to $F_1^{B\to K^*}(0)/F_1^{B\to\rho}(0) \simeq (1.14 \pm 0.02)$.

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1 Introduction

With the advent of the heavy-quark symmetry [1], there has been considerable interest and progress in the understanding of the semileptonic form factors of the transition of a heavy quark into another heavy quark, since in this infinite mass limit all semileptonic form factors reduce to the single Isgur-Wise function [2]. In the case of heavy-to-light transitions, Isgur and Wise [3] have remarked that, if one assumes that only the upper two components of the b quark field contribute, there will be relations between various form factors of the semileptonic and rare B-decays. The relations obtained from the static heavy quark approach are not rigorous as the momentum transfer of e.g, the $B \to K^*\gamma$, which is of the order of $M_b/2$, is large, in such a way that perturbative contributions related to the hard process [4] can invalidate the Isgur Wise relations. Burdman and Donoghue [5] have shown that in these processes, the soft quark model contributions dominate over the perturbative ones, such that the results of Isgur-Wise might still be valid and could be extended to the whole range of q^2.

In this paper, we shall examine the validity of the previous results, using the approach of QCD Spectral Sum Rules (QSSR) in the analysis of the form factor for the $B \to K^*\gamma$ radiative process. The form factor is defined as:

$$< K^*(p')|\bar{s}\sigma_{\mu\nu}\left(\frac{1+\gamma_5}{2}\right)q^\nu b|B(p) >= i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho p'^\sigma F_1^{B\to K^*}$$
$$+\left\{\epsilon^*_\mu(M_B^2-M_{K^*}^2)-\epsilon^* q(p+p')_\mu\right\}\frac{F_1^{B\to K^*}}{2}. \tag{1}$$

In the QSSR evaluation of the form factor , we shall consider the generic three-point function (omitting Lorentz indices):

$$V(p,p',q) = -\int d^4x \int d^4y \exp(ip'x - ipy) < 0|TJ_q(x)O(0)J_b(y)|0 >, \tag{2}$$

whose Lorentz decompositions are analogous to the previous hadronic amplitudes; $J_q \equiv \bar{s}\gamma^\mu d$ is the bilinear quark current having the quantum numbers of the K^*; $J_b \equiv (M_b + m_d)\bar{d}(i\gamma_5)b$ is the quark current associated to the B-meson; $O \equiv \bar{b}\frac{1}{2}\sigma_{\mu\nu}q^\nu s$ is the weak operator. The vertex function obeys the double dispersion relation:

$$V(p^2,p'^2,q^2) = \frac{1}{\pi^2}\int_{M_b^2}^\infty \frac{ds}{s-p^2}\int_0^\infty \frac{ds'}{s'-p'^2}\,\mathrm{Im}\,V(s,s',q^2), \tag{3}$$

where $q \equiv p - p'$ is the momentum transfer.

2 Choice of the sum rule

Widely used in the literature is the double exponential Laplace (Borel) sum rule (DLSR), which has given successful predictions in the heavy-to-heavy transitions. This method has been used also for the heavy-to-light transitions [6]–[9]. Though at finite b-quark mass value, the numerical fits of the form factors may be quite good, one can notice that the OPE in the DLSR blows up for $M_b \to \infty$, which invalidates the uses of the DLSR in this

channel. This is due to the existence of terms of the type:

$$\frac{M_b^{2l}}{\left(M_b^2-p^2\right)^k p'^{2k'}}, \tag{4}$$

which appear in the successive evaluation of the Wilson coefficients of high-dimension operators. After the double Laplace (Borel) transform, these terms convert into:

$$M_b^{2l}\tau^k\tau'^{k'}\exp(-\tau M_b^2), \tag{5}$$

where τ (resp. τ') is the sum-rule variable associated to the heavy (resp. light) quark. In this limit, it is convenient to introduce the non-relativistic sum-rule variable $\tau_{NR} \equiv M_b\tau$, where τ_{NR} is M_b-independent. Then, it is clear from (5) that the OPE converges *if and only if* $k \geq 2l$. This condition is not fulfilled in the case of the $B \to K^*\,\gamma$ process, as also noticed by [10], [9], while we have also checked that it is not satisfied in some of the form factors of the $B \to \rho$ and $D \to K^*$ semileptonic processes analyzed in [6],[7], though this does not effect the numerical estimates of these form factors. In order to restore the good behaviour of the OPE, the authors of Ref. [9] modelize the condensates in an *ad hoc* way. We do not find this argument convincing. Moreover, this parametrization cannot guarantee the convergence of the OPE at higher orders in dimensions.

An alternative promising direction is to find a sum rule, other than DLSR, which is more appropriate for this channel. The light-cone sum rule has been chosen in Ref. [10]; but, due to the complexity (and, perhaps, less understood structure) of the wave functions entering in this sum rule, we do not have here the simplicity and the transparency of the original SVZ-expansion, where, in this case, the non-perturbative effects are simulated by the vacuum condensates.

In this paper, we shall present another type of sum rule, which is in the line of the SVZ-expansion and which is adequate for the heavy-to-light process, since it has a good behaviour when $M_Q \to \infty$. This sum rule has been invented in [11] for the analysis of the $B \to D$, π and ρ semileptonic decays. This sum rule combines the moment (finite number of derivatives evaluated at $p^2 = 0$) which is good for the heavy quark as $M_b \gg \Lambda$ ($1/M_b$-expansion), and the Laplace one, which is appropriate for the light quark if $p'^2 \gg \Lambda^2$ ($1/p'^2$-expansion). We shall hereafter call this sum rule, as in [11], the *hybrid sum rule* (HSR). It has the form:

$$\begin{aligned}\mathcal{H}(\backslash,\tau') &\equiv \frac{1}{n!}\left(\frac{\partial}{\partial p^2}\right)^n_{p^2=0}\mathcal{L}\left(\mathcal{V}(\sqrt{}^{\in},\sqrt{}'^{\in},\Pi^{\in})\right)\\ &= \frac{1}{\pi^2}\int_{M_b^2}^{\infty}\frac{ds}{s^{n+1}}\int_0^{\infty}ds'\exp(-\tau's')\mathrm{Im}\,V(s,s',q^2),\end{aligned} \tag{6}$$

where $\mathcal{L}$ is the Laplace (Borel) exponential operator. As one can obviously notice, the *hybrid* transform of (4) is:

$$\frac{\tau'^{k'}}{M_b^{2(k+n-l)}}, \tag{7}$$

which shows that the convergence of the OPE for $M_b \to \infty$ is reached with the much weaker condition $k \geq l-n$ than the one $k \geq 2l$ of the DLSR. Indeed, in the specific case of the $B \to K^*\,\gamma$ process, the truncated series with the inclusion of the mixed condensate already converges for $n = 0$.

In order to come to observables, we insert intermediate states between the electromagnetic and hadronic currents in (2), while we smear the higher-states effects with the discontinuity of the QCD graphs from a threshold t_c (t'_c) for the heavy (light) mesons. Therefore, we have the sum rule for the form factor $F_1^{B\to V}(q^2)$:

$$\begin{aligned}\mathcal{H}_{\nabla\rceil\int} &\simeq 2C_V f_B \frac{F_1^{B\to V}(q^2)}{M_B^{2n}} \exp\left(-M_V^2\tau\right) \\ &\simeq \frac{1}{\pi^2}\int_{M_b^2}^{t_c}\frac{ds}{s^{n+1}}\int_0^{t'_c} ds' \exp(-\tau s') \mathrm{Im}\, V_{PT}(s,s',q^2) + \mathrm{NPT}. \qquad (8)\end{aligned}$$

PT (NPT) refers to perturbative (non perturbative) contributions; $C_V \equiv M_V^2/(2\gamma_V)$ for light vector mesons; M_V is the light-meson mass. The decay constants are normalized as:

$$\begin{aligned}(m_q + M_Q) < 0|\bar{q}(i\gamma_5)Q|P >= \sqrt{2}M_P^2 f_P \\ < 0|\bar{q}\gamma_\mu Q|V >= \epsilon_\mu^* \sqrt{2}\frac{M_V^2}{2\gamma_V}. \qquad (9)\end{aligned}$$

3 HSR estimate of the $B \to \rho\,\gamma$ form factor

In the following, we shall use the previous sum rule for the estimate of the $B \to \rho\,\gamma$ form factor. The QCD expression of the corresponding vertex function reads:

$$\begin{aligned}\mathrm{Im}V(s,s') &= \frac{3}{8}\frac{s' M_b^5}{(s-s')^3} \\ V_{qq} &= -\frac{M_b^2}{2}\frac{<\bar{d}d>}{(M_b^2-p^2)(-p'^2)}. \qquad (10)\end{aligned}$$

We shall use the contribution of the mixed condensate obtained by [10]. Then, we deduce the sum rule:

$$\begin{aligned}\mathcal{H}_{\nabla\rceil\int} &\simeq \frac{3}{8\pi^2}M_b^5\int_{M_b^2}^{t_c}\frac{ds}{s^{n+1}}\int_0^{t'_c} ds' \frac{s'}{(s-s')^3}\exp(-\tau s') \\ &- \frac{<\bar{d}d>(\tau')}{2M_b^{2n}}\left\{1-\tau' M_0^2\left(\frac{(n+1)}{3}+\frac{\tau'^{-1}}{4M_b^2}\left(n^2+3n+4\right)\right)\right\}. \qquad (11)\end{aligned}$$

We shall also introduce an analogous expression of the decay constant f_B from moments sum rule at the same order [18]:

$$\frac{2f_B^2}{(M_B^2)^{n-1}} \simeq \frac{3}{8\pi^2}M_b^2\int_{M_b^2}^{t_c}\frac{ds}{s^{n+1}}\frac{(s-M_b^2)^2}{s} - \frac{<\bar{d}d>}{M_b^{2n-1}}\left\{1-\frac{n(n+1)}{4}\left(\frac{M_0^2}{M_b^2}\right)\right\}. \qquad (12)$$

For convenience, we shall work with the non-relativistic energy parameters E and $\delta M_{(b)}$:

$$s \equiv (M_b+E)^2, \quad \delta M_{(b)} \equiv M_B - M_b, \qquad (13)$$

where, as we have seen in the analysis of the two-point correlator, the continuum energy E_c is [18], [19]:

$$\begin{aligned}E_c^D &\simeq (1.08 \pm 0.26)\ \mathrm{GeV}, \\ E_c^B &\simeq (1.30 \pm 0.10)\ \mathrm{GeV}, \\ E_c^\infty &\simeq (1.50 \sim 1.70)\ \mathrm{GeV}. \qquad (14)\end{aligned}$$

Using (8), (10)–(14), we calculate the form factor $F_1^B(0)$ in Fig. 1 for different values of n, τ', E_c and t'_c. We use the following values of the QCD parameters for 5 flavours [12, 13]:

$$\begin{aligned} M_0^2 &= (0.8 \pm 0.1) \text{ GeV}^2, \qquad \Lambda \; = \; (175 \pm 41) \text{ MeV}, \\ M_b &= (4.59 \pm 0.05) \text{ GeV}, \quad\; M_c = (1.47 \pm 0.05) \text{ GeV}, \\ < \bar{d}d > (\tau') &= -(189 \text{ MeV})^3 \left(-\log \tau'^{1/2}\Lambda\right)^{12/23}, \end{aligned} \tag{15}$$

and the experimental value $\gamma_\rho = (2.55 \pm 0.06)$ from the ρ-meson electronic width. Values of τ' and t'_c at which this experimental number is reproduced from the ρ sum rule are [12]: $\tau' \simeq (0.6 \sim 1.0)\,\text{GeV}^{-2}$ and $t'_c \simeq (1.7 \pm 0.3)\,\text{GeV}^2$. As can be noticed from Fig. 1, the values of τ' corresponding to the stability are consistent with the previous ones. Increasing values of n tend to destroy the existence of the stability points due to the increase of the anomalously large values of the $1/M_b^2$ contributions. The inclusion of higher-dimension condensates in the OPE should restore the stability for larger values of n. In our present truncated series, the different effects remain still corrections to the $< \bar{q}q >$ condensate ones, for $n \leq 2$. Moving the values of E_c^B and t'_c within the previous ranges modifies the shape of the curves but affects only slightly the value of the stability point; Λ and M_b introduce each an error of 4 and 2%. We do not expect that radiative corrections will affect this result in a sensible way from different experiences of calculating similar observables in the heavy-to-heavy transition form factors. Indeed, large radiative corrections due to the coulombic-like interactions cancel out in the ratio of the three- over the two-point function sum rules while the radiative corrections due to the light quark condensate is known to be small in the cases of heavy and of light quark processes. In these cases, the total effect due to the radiative corrections is about 4-5%, which we consider as another source of errors. Taking into account these different sources of errors, we obtain from the HSR:

$$F_1^{B\to\rho}(0) \simeq (27.0 \pm 1.1 \pm 3.2) \times 10^{-2}, \tag{16}$$

where the first error comes from the sum-rule procedure and the second one from the input parameters. Extending this analysis to the D mass, we get, for the hypothetical $D \to \rho\,\gamma$ process:

$$F_1^{D\to\rho}(0) \simeq (62.0 \pm 10.0) \times 10^{-2}. \tag{17}$$

4 M_b-dependence of the $B \to \rho\,\gamma$ form factor

In order to understand the meaning of the previous results, let us study *analytically* the sum rule at large values of M_b. Since we shall work with the full theory of QCD, the pseudoscalar quark current associated to the B meson does not acquire any anomalous dimension. Using [18]

$$\begin{aligned} f_B^2 \;\simeq\; & \frac{1}{\pi^2}\frac{\left(E_c^B\right)^3}{M_b}\left(\frac{M_B}{M_b}\right)^{2n-1}\Bigg\{(1-\frac{3}{2}(n+1)\left(\frac{E_c^B}{M_b}\right)+\frac{3}{5}\left((2n+3)(n+1)+\frac{1}{4}\right)\left(\frac{E_c^B}{M_b}\right)^2 \\ & - \frac{\pi^2}{2}\frac{< \bar{d}d >}{\left(E_c^B\right)^3}\left(1-\frac{n(n+1)}{4}\frac{M_0^2}{M_b^2}\right)\Bigg\}, \end{aligned} \tag{18}$$

from (12), we can deduce from (11):

$$F_1^{B\rightarrow\rho}(0) \simeq -\frac{\pi}{4}\frac{\sqrt{M_b}}{(E_c^B)^{3/2}}\frac{<\bar{d}d>}{C_\rho}\exp(M_\rho^2\tau')\left(1+\delta^{(0)}+\frac{\delta^{(1)}}{M_b}+\frac{\delta^{(2)}}{M_b^2}\right), \tag{19}$$

with:

$$\begin{aligned}
\delta^{(0)} &= -\frac{(n+1)}{6}M_0^2\tau' + \frac{\pi^2}{4}\frac{<\bar{q}q>}{(E_c^B)^3} \\
\delta^{(1)} &= \frac{3}{4}(n+1)E_c^B + \left(n+\frac{1}{2}\right)\delta M_{(b)} \\
\delta^{(2)} &= -\frac{2\mathcal{I}}{<\bar{d}d>} - \frac{M_0^2}{4}\left(1+n(n+1)\frac{\pi^2}{4}\frac{<\bar{q}q>}{(E_c^B)^3}\right) + \frac{3\left(E_c^B\right)^2}{640}(83n^2+230n+163),
\end{aligned} \tag{20}$$

where:

$$\mathcal{I} \equiv \frac{\ni}{\forall\pi^{\in}}\int_{\prime}^{\mathcal{E}_\rceil^{\mathcal{B}}} \frac{\in\lceil\mathcal{E}}{\left(\infty+\frac{\mathcal{E}_\rceil^{\mathcal{B}}}{\mathcal{M}_\lfloor}\right)^{\in\backslash+\infty}} \int_{\prime}^{\sqcup'_\rceil} \lceil\int' \frac{\int'}{\left(\left(\infty+\frac{\mathcal{E}}{\mathcal{M}_\lfloor}\right)^{\in}-\int'\right)^{\ni}} \exp(-\tau\int'). \tag{21}$$

We have checked that this approximate expression gives a slightly lower (about 20%) value of $F_1^{B\rightarrow\rho}(0)$.

We evaluate *numerically* the coefficients of the $1/M_b$ and $1/M_b^2$ terms at the values $n \simeq 0 \sim 1$ and $\tau' \simeq 0.5 \sim 0.7$ GeV^{-2}, where the HSR optimizes, from the full non-expanded expression of the three-point function. We use the expression of f_B given by (18), which naturally has the expected large M_b behaviour. Then, we deduce the interpolating formula in units of GeV:

$$F_1^{B\rightarrow\rho}(0) \simeq -10.5\ \text{GeV}^{-2}\sqrt{M_b}\frac{<\bar{d}d>(\tau')}{(E_c^B)^{3/2}}\left\{1+\frac{2.5\pm1.1}{M_b}+\frac{6.3\pm1.1}{M_b^2}\right\}, \tag{22}$$

where each coefficient compares reasonably well with that of the expanded expression. We have absorbed the error due to E_c^B into the errors in the corrections. One should understand in the previous formula that:

The overall factor 10.5 is fixed in such a way that the interpolating formula reproduces, with the central values of the numbers in (22), the numerical estimate in (16). This factor also absorbs in it the effect of the mixed condensate M_0^2 as given by $\delta^{(0)}$. However, if we assume that the factorisation works for the high-dimension condensates (however, it is known to be largely violated by a factor 2 to 3 [12]), we could resum all light-quark-like condensates effects (idea behind the notion of non-local condensate) by replacing $<\bar{d}d>$ with $<\bar{d}d>\exp(-M_0^2\tau'/3)$, which converges perfectly for $M_b\rightarrow\infty$ contrary to the case of the DLSR mentioned by [10].

The $1/M_b$ correction is mainly due to f_B (one should compare this coefficient with the one of f_B including the $1/M_b^2$ term (see e.g. [11]))and to the meson-quark mass-difference $\delta M_{(b)}$ (see (20)).

The coefficient of the $1/M_b^2$ term comes, partly from an $1/M_b$-expansion of f_B, which is known to give a quite good approximation of f_B even at the c quark mass. However,

the main contribution to the $1/M_b^2$ term in (22) comes from the perturbative vertex diagram. One should understand that the extraction of this effect comes from the exact non-expanded expression of the Wilson coefficient *without any approximation* related to the large value of M_b. The appearance of the $1/M_b^2$ term is only due to the analytical structure of the perturbative contribution, and its numerical coefficient absorbs in it all the effects of the perturbative graph. From this feature, the dominance of this term at the c quark mass does not mean that the formula given in (22) cannot be used at this scale. One should understand (22) as an *interpolating* formula.

This sum rule explicitly indicates that the M_b behaviour is dominated by the soft process term $< \bar{q}q >$ instead of the perturbative hard diagram for $M_b \rightarrow \infty$. This is a peculiar feature of this heavy-to-light transition process, which is not the case of the heavy-to-heavy one. We also obtain a similar behaviour in the case of the semileptonic $B \rightarrow \pi\ e\nu$ form factor [14]. However, the authors of Refs.[10], [15] who work with the light-cone sum rule do not have this dominant behaviour in M_b, since in their case the behaviour $M_b^{-3/2}$ is similar to that coming from the perturbative graph, which is non-leading in M_b, in our approach based on the SVZ-expansion. A clarification of this disrepancy needs a better understanding of the structure of the meson wave functions used in this analysis for $M_b \rightarrow \infty$. The M_b dependence obtained here at $q^2 = 0$ is the same as the one obtained by Isgur-Wise at q^2_{max}, which might be in line with the Isgur-Wise conjecture that the relations among form factors can be valid at any q^2 values if the heavy-to-light transition form factors are dominated by the soft process: in such a case, the heavy quark stays almost on its mass shell. The dominance of the soft process obtained by Burdman-Donoghue [5] is confirmed, in a completely independent way, by our analysis.

However, at the real value of M_b, the agreement of the different previous sum-rule predictions for the B and D meson form factors is encouraging, despite the fact that the sum rule used in [9] is not well-defined for $M_b \rightarrow \infty$ (however, truncating their QCD series at the level of the quark condensate, one can notice that their result has the same M_b-behaviour than ours (see their equation (24)), while the results of [10] have a similar M_b-behaviour than our non-leading perturbative contribution. The agreement between different numerical estimates might be due to the large numerical value of the $M_b^{-3/2}$-term, which can also invalidate the naïve extrapolation of the result from the D to the B when only the leading M_b behaviour of the form factor is used. Using the previous interpolating formula at the D mass, we obtain:

$$F_1^{D \rightarrow \rho}(0) \simeq (61.5 \pm 30.5) \times 10^{-2}, \tag{23}$$

in accordance with the previous result in (17) from a numerical fit.

5 $SU(3)_f$-breakings and the $B \rightarrow K^*\ \gamma$ process

We shall consider in this section the explicit $SU(3)_f$-breakings on the form factor of the $B \rightarrow K^*\ \gamma$, process, due to the s quark mass and to the $< \bar{s}s >$ condensate, which have the values [12, 13]:

$$\bar{m}_s(\tau') \simeq 150 \text{ MeV} \qquad \frac{< \bar{s}s >}{< \bar{d}d >} \simeq (0.6 \pm 0.1). \tag{24}$$

For this process, the QCD expression of the $SU(3)_f$ breaking parts of the vertex function reads to leading order in α_s:

$$\begin{aligned} \mathrm{Im}V(s,s')_{SU(3)} &\rightarrow \frac{3}{8}M_b^2 m_s \frac{s-s'-M_b^2}{(s-s')^2} \\ <\bar{q}q>_{SU(3)} &\rightarrow -\frac{m_s}{2}M_b <\bar{d}d> . \end{aligned} \tag{25}$$

One can notice that the only effect of the quark condensate which survives after the sum rule procedure is the one from the d quark line of the B-meson current, such that only the $<\bar{d}d>$ condensate contributes. Using an analytical approximate evaluation of the $SU(3)_f$-breaking effects, we deduce the K^* version of the interpolating formula in (22):

$$\begin{aligned} F_1^{B\rightarrow K^*}(0) &\simeq -11.3 \text{ GeV}^{-2}\sqrt{M_b}\frac{<\bar{d}d>(\tau')}{(E_c^B)^{3/2}} \\ & \left\{1+\frac{\bar{m}_s}{M_b}+\frac{2.5\pm1.0}{M_b}+\frac{6.3\pm1.1}{M_b^2}\left(1+2\frac{\bar{m}_s E_c^B}{t'_c}\right)\right\}, \end{aligned} \tag{26}$$

where we have used $\gamma_{K^*} \simeq (2.80 \pm 0.13)$ from τ decay. One should notice that the $SU(3)_f$-breakings due to the meson mass and coupling give an effect of +7.7 %, which is included in the overall factor. The explicit breaking due to the s quark mass from the Wilson coefficient of the condensate contributes as +3.0%. The $SU(3)$ breaking from the perturbative diagram is about 23% of the perturbative contribution and might explain the large $SU(3)_f$-breakings obtained in [10].

One can also note that the $SU(3)_f$-breakings vanish for $M_b \rightarrow \infty$, contrary to the case of f_{B_s} where these corrections remain constant [20]. We deduce from the previous formula:

$$F_1^{B\rightarrow K^*}(0)/F_1^{B\rightarrow\rho}(0) \simeq (1.14\pm0.02), \quad F_1^{D\rightarrow K^*}(0)/F_1^{D\rightarrow\rho}(0) \simeq (1.22\pm0.04), \tag{27}$$

where it is clear that the systematic errors in the evaluation of the coefficients of the $1/M_b$ and $1/M_b^2$ cancel out in the ratio. The quoted error in (27) is mainly due to $SU(3)_f$-breaking from the perturbative graph. The $SU(3)_f$-breaking corrections are smaller than the ones in Ref. [10] as explained before. Combining this result with the one in (16), we deduce:

$$F_1^{B\rightarrow K^*} \simeq (30.8\pm1.3\pm3.6\pm0.6)\times10^{-2}, \tag{28}$$

where the last error is due to (27). This implies the branching ratio:

$$Br(B\rightarrow K^*\,\gamma) \simeq (4.45\pm1.12)\times10^{-5}, \tag{29}$$

in agreement with the CLEO data of $(4.5\pm1.5\pm0.9)\times10^{-5}$ [16]. Our result for $F_1^{B\rightarrow K^*}(0)$ is in agreement with the one obtained from an effective lagrangian approach [17]. It also agrees with the value (0.32 ± 0.05) obtained from the light-cone sum rule [10], which, *a priori*, is an approach quite different from ours, as indeed, the two approaches do not provide the same large M_b-behaviour of the form factor. A comparison with the value (0.35 ± 0.05) obtained in [9] from the DLSR is not very informative as this number comes for the uses of inconsistent sets of input parameters as noticed by [10] (we agree with these criticisms). Moreover, the analysis of [9] also suffers from the bad behaviour of the DLSR which blows up for large M_b.

6 Conclusions

We have estimated the $B \to \rho\ \gamma$ form factor in (16). The value of the hypothetical $D \to \rho\ \gamma$ form factor is given in (17). The value of the $B \to K^*\ \gamma$ form factor including $SU(3)_f$-breakings is given in (28) and leads to the branching ratio in (29).

We have performed our analysis with the so-called *hybrid* sum rule (HSR) in (6), which is well-defined for $M_b \to \infty$, contrary to the case of the double exponential Laplace sum rule (DLSR) which blows up in this peculiar process. Our numerical estimates are in agreement with the previous results from light-cone sum rule [10], though the two results do not have the same large M_b-behaviour.

Indeed, we have also studied the M_b-dependence of the previous transition form factor which can be parametrized with the interpolating formula in (24). It explicitly shows that, in the large-mass limit, this form factor is dominated by the light quark condensate and behaves like $\sqrt{M_b}$, though at low M_b mass the perturbative contribution is *numerically* important. This dominance of the soft $< \bar{d}d >$ contribution, which is in line with the results of Isgur-Wise [3] and Donoghue-Burdman [5], might allow the extension, of the result obtained at q^2_{max}, to the whole range of q^2-values. In this large mass limit, the light-cone sum rule result has a $M_b^{-3/2}$ behaviour, which is very similar to the one from the perturbative diagram, in our approach within the SVZ-expansion. A clarification of this discrepancy between our result with the one from the light-cone, needs a better understanding of the meaning of the $< \bar{q}q >$ condensate in the language of the wave functions.

Finally, we have extracted an analytic expression of the $SU(3)_f$-breaking terms due to the s quark mass in (26) and (27). It shows, that the $SU(3)_f$-breakings tend to zero for $M_b \to \infty$ in contrast with the case of f_{B_s}[20]. Our numerical value in (27) is smaller than the one in [10].

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Figure captions

τ' and n dependences of the form factor $F_1^{B\to\rho}(0)$ of the $B \to \rho\ \gamma$ process.

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